

```

1 *SAVE K4 2 113 / Solving mod(X^4+Y^2,113)=0 for 0<=X,Y<=113
2 *LOAD INDEX
3 *
4 *Formulas:
5 *P=113
6 *I(X,Y):=(15*Y-X^2+P*X)/P
7 *J(X,Y):=(15*Y+X^2-P*X)/P
8 *
9 *HEADER= FRAME=1
10 *
11 *XSCALE=[Swiss(50)],0,20,40,60,80,113
12 *YSCALE=[Swiss(50)],0,20,40,60,80,113
13 *
14 *XSCALE=0,113 YSCALE=0,113
15 *WSTYLE=0 HEADER= XLABEL= YLABEL= XDIV=1,20,1 YDIV=1,20,1 FRAME=3
16 *WSIZE=990,990 WHOME=500,45 POINT=6,6 MODE=2000,2000
17 *
18 *.....
19 *
20 *Auxiliary sucros for manipulating the table of results:
21 *TUTSAVE A
22 + A: {R}{next word}{save word W1}
23 - if W1 '=' {empty} then goto E
24 *{l}&{goto A}
25 + E: {end}
26 *
27 *TUTSAVE ZZ / for finding type M (1 or 2) of roots
28 /
29 + A: {R}
30 *{save word W1}
31 - if W1 '=' {sp} then goto E
32 *{save word W1}{next word}{save word W2}
33 *{line start}{r11}I({write W1},{write W2})={act}{line end}
34 * J({write W1},{write W2})={act}{goto A}
35 + E: {end}
36 *
37 *Finding roots X,Y:
38 *DIOPH 4,2,113,113 / / n_comb=204
39 *
40 *Columns M,L from I and J values manually
41 *
42 *DATA XYML
43 *X Y M L
44 *0 0 1 0 I(0,0)=0 J(0,0)=0
45 *0 113 2 15 I(0,113)=15 J(0,113)=15
46 *1 15 2 1 I(1,15)=2.9823008849558 J(1,15)=1
47 *1 98 1 14 I(1,98)=14 J(1,98)=12.017699115044
48 *2 53 1 9 I(2,53)=9 J(2,53)=5.070796460177
49 *2 60 2 6 I(2,60)=9.929203539823 J(2,60)=6
50 *3 22 2 0 I(3,22)=5.8407079646018 J(3,22)=0
51 *3 91 1 15 I(3,91)=15 J(3,91)=9.1592920353982
52 *4 14 1 -2 I(4,14)=5.716814159292 J(4,14)=-2
53 *4 99 1 17 I(4,99)=17 J(4,99)=9.283185840708
54 *5 36 2 0 I(5,36)=9.5575221238938 J(5,36)=0
55 *5 77 1 15 I(5,77)=15 J(5,77)=5.4424778761062
56 *6 25 1 9 I(6,25)=9 J(6,25)=-2.3628318584071
57 *6 88 2 6 I(6,88)=17.362831858407 J(6,88)=6
58 *7 56 1 14 I(7,56)=14 J(7,56)=0.86725663716814
59 *7 57 2 1 I(7,57)=14.132743362832 J(7,57)=1
60 *8 56 2 0 I(8,56)=14.867256637168 J(8,56)=0
61 *8 57 2 25 I(8,57)=15 J(8,57)=0.13274336283186
62 *9 28 2 23 I(9,28)=12 J(9,28)=-4.5663716814159
63 *9 85 2 3 I(9,85)=19.566371681416 J(9,85)=3
64 *10 31 1 -5 I(10,31)=13.230088495575 J(10,31)=-5
65 *10 82 1 20 I(10,82)=20 J(10,82)=1.7699115044248
66 *11 7 2 -9 I(11,7)=10.858407079646 J(11,7)=-9
67 *11 106 1 24 I(11,106)=24 J(11,106)=4.141592920354
68 *12 13 2 -9 I(12,13)=12.451327433628 J(12,13)=-9
69 *12 100 1 24 I(12,100)=24 J(12,100)=2.5486725663717
70 *13 49 2 -5 I(13,49)=18.008849557522 J(13,49)=-5
71 *13 64 1 20 I(13,64)=20 J(13,64)=-3.0088495575221
72 *14 2 2 -12 I(14,2)=12.530973451327 J(14,2)=-12
73 *14 111 1 27 I(14,111)=27 J(14,111)=2.4690265486726
74 *15 15 2 15 I(15,15)=15 J(15,15)=-11.017699115044
75 *15 98 2 0 I(15,98)=26.017699115044 J(15,98)=0

```

76	*16	2	1	14	I(16,2)=14	J(16,2)=-13.469026548673
77	*16	111	2	1	I(16,111)=28.469026548673	J(16,111)=1
78	*17	41	2	-9	I(17,41)=19.884955752212	J(17,41)=-9
79	*17	72	1	24	I(17,72)=24	J(17,72)=-4.8849557522124
80	*18	1	2	-15	I(18,1)=15.265486725664	J(18,1)=-15
81	*18	112	1	30	I(18,112)=30	J(18,112)=-0.26548672566372
82	*19	9	1	17	I(19,9)=17	J(19,9)=-14.610619469027
83	*19	104	2	-2	I(19,104)=29.610619469027	J(19,104)=-2
84	*20	11	2	-15	I(20,11)=17.920353982301	J(20,11)=-15
85	*20	102	1	30	I(20,102)=30	J(20,102)=-2.9203539823009
86	*21	52	1	24	I(21,52)=24	J(21,52)=-10.194690265487
87	*21	61	2	-9	I(21,61)=25.194690265487	J(21,61)=-9
88	*22	28	2	-14	I(22,28)=21.433628318584	J(22,28)=-14
89	*22	85	1	29	I(22,85)=29	J(22,85)=-6.4336283185841
90	*23	25	2	-15	I(23,25)=21.637168141593	J(23,25)=-15
91	*23	88	1	30	I(23,88)=30	J(23,88)=-6.6371681415929
92	*24	52	2	-12	I(24,52)=25.805309734513	J(24,52)=-12
93	*24	61	1	27	I(24,61)=27	J(24,61)=-10.805309734513
94	*25	4	1	20	I(25,4)=20	J(25,4)=-18.938053097345
95	*25	109	2	-5	I(25,109)=33.938053097345	J(25,109)=-5
96	*26	30	1	24	I(26,30)=24	J(26,30)=-16.035398230088
97	*26	83	2	-9	I(26,83)=31.035398230088	J(26,83)=-9
98	*27	26	1	24	I(27,26)=24	J(27,26)=-17.097345132743
99	*27	87	2	-9	I(27,87)=32.097345132743	J(27,87)=-9
100	*28	8	2	-20	I(28,8)=22.12389380531	J(28,8)=-20
101	*28	105	1	35	I(28,105)=35	J(28,105)=-7.1238938053097
102	*29	41	1	27	I(29,41)=27	J(29,41)=-16.115044247788
103	*29	72	2	-12	I(29,72)=31.115044247788	J(29,72)=-12
104	*30	53	2	-15	I(30,53)=29.070796460177	J(30,53)=-15
105	*30	60	1	30	I(30,60)=30	J(30,60)=-14.070796460177
106	*31	49	1	29	I(31,49)=29	J(31,49)=-15.991150442478
107	*31	64	2	-14	I(31,64)=30.991150442478	J(31,64)=-14
108	*32	8	1	24	I(32,8)=24	J(32,8)=-21.87610619469
109	*32	105	2	-9	I(32,105)=36.87610619469	J(32,105)=-9
110	*33	50	1	30	I(33,50)=30	J(33,50)=-16.725663716814
111	*33	63	2	-15	I(33,63)=31.725663716814	J(33,63)=-15
112	*34	51	2	-17	I(34,51)=30.53982300885	J(34,51)=-17
113	*34	62	1	32	I(34,62)=32	J(34,62)=-15.53982300885
114	*35	44	1	30	I(35,44)=30	J(35,44)=-18.318584070796
115	*35	69	2	-15	I(35,69)=33.318584070796	J(35,69)=-15
116	*36	4	2	-24	I(36,4)=25.061946902655	J(36,4)=-24
117	*36	109	1	39	I(36,109)=39	J(36,109)=-10.061946902655
118	*37	31	1	29	I(37,31)=29	J(37,31)=-20.769911504425
119	*37	82	2	-14	I(37,82)=35.769911504425	J(37,82)=-14
120	*38	36	1	30	I(38,36)=30	J(38,36)=-20.442477876106
121	*38	77	2	-15	I(38,77)=35.442477876106	J(38,77)=-15
122	*39	11	1	27	I(39,11)=27	J(39,11)=-24.079646017699
123	*39	102	2	-12	I(39,102)=39.079646017699	J(39,102)=-12
124	*40	44	2	-20	I(40,44)=31.681415929204	J(40,44)=-20
125	*40	69	1	35	I(40,69)=35	J(40,69)=-16.681415929204
126	*41	16	2	-24	I(41,16)=28.247787610619	J(41,16)=-24
127	*41	97	1	39	I(41,97)=39	J(41,97)=-13.247787610619
128	*42	18	2	-24	I(42,18)=28.778761061947	J(42,18)=-24
129	*42	95	1	39	I(42,95)=39	J(42,95)=-13.778761061947
130	*43	50	2	-20	I(43,50)=33.274336283186	J(43,50)=-20
131	*43	63	1	35	I(43,63)=35	J(43,63)=-18.274336283186
132	*44	1	1	27	I(44,1)=27	J(44,1)=-26.734513274336
133	*44	112	2	-12	I(44,112)=41.734513274336	J(44,112)=-12
134	*45	22	1	30	I(45,22)=30	J(45,22)=-24.159292035398
135	*45	91	2	-15	I(45,91)=39.159292035398	J(45,91)=-15
136	*46	13	1	29	I(46,13)=29	J(46,13)=-25.548672566372
137	*46	100	2	-14	I(46,100)=40.548672566372	J(46,100)=-14
138	*47	26	2	-24	I(47,26)=30.902654867257	J(47,26)=-24
139	*47	87	1	39	I(47,87)=39	J(47,87)=-15.902654867257
140	*48	18	1	30	I(48,18)=30	J(48,18)=-25.221238938053
141	*48	95	2	-15	I(48,95)=40.221238938053	J(48,95)=-15
142	*49	32	1	32	I(49,32)=32	J(49,32)=-23.504424778761
143	*49	81	2	-17	I(49,81)=38.504424778761	J(49,81)=-17
144	*50	16	1	30	I(50,16)=30	J(50,16)=-25.752212389381
145	*50	97	2	-15	I(50,97)=40.752212389381	J(50,97)=-15
146	*51	30	2	-24	I(51,30)=31.964601769912	J(51,30)=-24
147	*51	83	1	39	I(51,83)=39	J(51,83)=-16.964601769912
148	*52	7	1	29	I(52,7)=29	J(52,7)=-27.141592920354
149	*52	106	2	-14	I(52,106)=42.141592920354	J(52,106)=-14
150	*53	14	1	30	I(53,14)=30	J(53,14)=-26.283185840708
151	*53	99	2	-15	I(53,99)=41.283185840708	J(53,99)=-15

152	*54	9	2	-27	I(54,9)=29.389380530973	J(54,9)=-27
153	*54	104	1	42	I(54,104)=42	J(54,104)=-14.389380530973
154	*55	51	1	35	I(55,51)=35	J(55,51)=-21.46017699115
155	*55	62	2	-20	I(55,62)=36.46017699115	J(55,62)=-20
156	*56	32	2	-24	I(56,32)=32.495575221239	J(56,32)=-24
157	*56	81	1	39	I(56,81)=39	J(56,81)=-17.495575221239
158	*57	32	2	-24	I(57,32)=32.495575221239	J(57,32)=-24
159	*57	81	1	39	I(57,81)=39	J(57,81)=-17.495575221239
160	*58	51	1	35	I(58,51)=35	J(58,51)=-21.46017699115
161	*58	62	2	-20	I(58,62)=36.46017699115	J(58,62)=-20
162	*59	9	2	-27	I(59,9)=29.389380530973	J(59,9)=-27
163	*59	104	1	42	I(59,104)=42	J(59,104)=-14.389380530973
164	*60	14	1	30	I(60,14)=30	J(60,14)=-26.283185840708
165	*60	99	2	-15	I(60,99)=41.283185840708	J(60,99)=-15
166	*61	7	1	29	I(61,7)=29	J(61,7)=-27.141592920354
167	*61	106	2	-14	I(61,106)=42.141592920354	J(61,106)=-14
168	*62	30	2	-24	I(62,30)=31.964601769912	J(62,30)=-24
169	*62	83	1	39	I(62,83)=39	J(62,83)=-16.964601769912
170	*63	16	1	30	I(63,16)=30	J(63,16)=-25.752212389381
171	*63	97	2	-15	I(63,97)=40.752212389381	J(63,97)=-15
172	*64	32	1	32	I(64,32)=32	J(64,32)=-23.504424778761
173	*64	81	2	-17	I(64,81)=38.504424778761	J(64,81)=-17
174	*65	18	1	30	I(65,18)=30	J(65,18)=-25.221238938053
175	*65	95	2	-15	I(65,95)=40.221238938053	J(65,95)=-15
176	*66	26	2	-24	I(66,26)=30.902654867257	J(66,26)=-24
177	*66	87	1	39	I(66,87)=39	J(66,87)=-15.902654867257
178	*67	13	1	29	I(67,13)=29	J(67,13)=-25.548672566372
179	*67	100	2	-14	I(67,100)=40.548672566372	J(67,100)=-14
180	*68	22	1	30	I(68,22)=30	J(68,22)=-24.159292035398
181	*68	91	2	-15	I(68,91)=39.159292035398	J(68,91)=-15
182	*69	1	1	27	I(69,1)=27	J(69,1)=-26.734513274336
183	*69	112	2	-12	I(69,112)=41.734513274336	J(69,112)=-12
184	*70	50	2	-20	I(70,50)=33.274336283186	J(70,50)=-20
185	*70	63	1	35	I(70,63)=35	J(70,63)=-18.274336283186
186	*71	18	2	-24	I(71,18)=28.778761061947	J(71,18)=-24
187	*71	95	1	39	I(71,95)=39	J(71,95)=-13.778761061947
188	*72	16	2	-24	I(72,16)=28.247787610619	J(72,16)=-24
189	*72	97	1	39	I(72,97)=39	J(72,97)=-13.247787610619
190	*73	44	2	-20	I(73,44)=31.681415929204	J(73,44)=-20
191	*73	69	1	35	I(73,69)=35	J(73,69)=-16.681415929204
192	*74	11	1	27	I(74,11)=27	J(74,11)=-24.079646017699
193	*74	102	2	-12	I(74,102)=39.079646017699	J(74,102)=-12
194	*75	36	1	30	I(75,36)=30	J(75,36)=-20.442477876106
195	*75	77	2	-15	I(75,77)=35.442477876106	J(75,77)=-15
196	*76	31	1	29	I(76,31)=29	J(76,31)=-20.769911504425
197	*76	82	2	-14	I(76,82)=35.769911504425	J(76,82)=-14
198	*77	4	2	-24	I(77,4)=25.061946902655	J(77,4)=-24
199	*77	109	1	39	I(77,109)=39	J(77,109)=-10.061946902655
200	*78	44	1	30	I(78,44)=30	J(78,44)=-18.318584070796
201	*78	69	2	-15	I(78,69)=33.318584070796	J(78,69)=-15
202	*79	51	2	-17	I(79,51)=30.53982300885	J(79,51)=-17
203	*79	62	1	32	I(79,62)=32	J(79,62)=-15.53982300885
204	*80	50	1	30	I(80,50)=30	J(80,50)=-16.725663716814
205	*80	63	2	-15	I(80,63)=31.725663716814	J(80,63)=-15
206	*81	8	1	24	I(81,8)=24	J(81,8)=-21.87610619469
207	*81	105	2	-9	I(81,105)=36.87610619469	J(81,105)=-9
208	*82	49	1	29	I(82,49)=29	J(82,49)=-15.991150442478
209	*82	64	2	-14	I(82,64)=30.991150442478	J(82,64)=-14
210	*83	53	2	-15	I(83,53)=29.070796460177	J(83,53)=-15
211	*83	60	1	30	I(83,60)=30	J(83,60)=-14.070796460177
212	*84	41	1	27	I(84,41)=27	J(84,41)=-16.115044247788
213	*84	72	2	-12	I(84,72)=31.115044247788	J(84,72)=-12
214	*85	8	2	-20	I(85,8)=22.12389380531	J(85,8)=-20
215	*85	105	1	35	I(85,105)=35	J(85,105)=-7.1238938053097
216	*86	26	1	24	I(86,26)=24	J(86,26)=-17.097345132743
217	*86	87	2	-9	I(86,87)=32.097345132743	J(86,87)=-9
218	*87	30	1	24	I(87,30)=24	J(87,30)=-16.035398230088
219	*87	83	2	-9	I(87,83)=31.035398230088	J(87,83)=-9
220	*88	4	1	20	I(88,4)=20	J(88,4)=-18.938053097345
221	*88	109	2	-5	I(88,109)=33.938053097345	J(88,109)=-5
222	*89	52	2	-12	I(89,52)=25.805309734513	J(89,52)=-12
223	*89	61	1	27	I(89,61)=27	J(89,61)=-10.805309734513
224	*90	25	2	-15	I(90,25)=21.637168141593	J(90,25)=-15
225	*90	88	1	30	I(90,88)=30	J(90,88)=-6.6371681415929
226	*91	28	2	-14	I(91,28)=21.433628318584	J(91,28)=-14
227	*91	85	1	29	I(91,85)=29	J(91,85)=-6.4336283185841

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228 *92 52 1 24 I(92,52)=24 J(92,52)=-10.194690265487
229 *92 61 2 -9 I(92,61)=25.194690265487 J(92,61)=-9
230 *93 11 2 -15 I(93,11)=17.920353982301 J(93,11)=-15
231 *93 102 1 30 I(93,102)=30 J(93,102)=-2.9203539823009
232 *94 9 1 17 I(94,9)=17 J(94,9)=-14.610619469027
233 *94 104 2 -2 I(94,104)=29.610619469027 J(94,104)=-2
234 *95 1 2 -15 I(95,1)=15.265486725664 J(95,1)=-15
235 *95 112 1 30 I(95,112)=30 J(95,112)=-0.26548672566372
236 *96 41 2 -9 I(96,41)=19.884955752212 J(96,41)=-9
237 *96 72 1 24 I(96,72)=24 J(96,72)=-4.8849557522124
238 *97 2 1 14 I(97,2)=14 J(97,2)=-13.469026548673
239 *97 111 2 1 I(97,111)=28.469026548673 J(97,111)=1
240 *98 15 1 15 I(98,15)=15 J(98,15)=-11.017699115044
241 *98 98 2 0 I(98,98)=26.017699115044 J(98,98)=0
242 *99 2 2 -12 I(99,2)=12.530973451327 J(99,2)=-12
243 *99 111 1 27 I(99,111)=27 J(99,111)=2.4690265486726
244 *100 49 2 -5 I(100,49)=18.008849557522 J(100,49)=-5
245 *100 64 1 20 I(100,64)=20 J(100,64)=-3.0088495575221
246 *101 13 2 -9 I(101,13)=12.451327433628 J(101,13)=-9
247 *101 100 1 24 I(101,100)=24 J(101,100)=2.5486725663717
248 *102 7 2 -9 I(102,7)=10.858407079646 J(102,7)=-9
249 *102 106 1 24 I(102,106)=24 J(102,106)=4.141592920354
250 *103 31 2 -5 I(103,31)=13.230088495575 J(103,31)=-5
251 *103 82 1 20 I(103,82)=20 J(103,82)=1.7699115044248
252 *104 28 1 12 I(104,28)=12 J(104,28)=-4.5663716814159
253 *104 85 2 3 I(104,85)=19.566371681416 J(104,85)=3
254 *105 56 2 0 I(105,56)=14.867256637168 J(105,56)=0
255 *105 57 1 15 I(105,57)=15 J(105,57)=0.13274336283186
256 *106 56 1 14 I(106,56)=14 J(106,56)=0.86725663716814
257 *106 57 2 1 I(106,57)=14.132743362832 J(106,57)=1
258 *107 25 1 9 I(107,25)=9 J(107,25)=-2.3628318584071
259 *107 88 2 6 I(107,88)=17.362831858407 J(107,88)=6
260 *108 36 2 0 I(108,36)=9.5575221238938 J(108,36)=0
261 *108 77 1 15 I(108,77)=15 J(108,77)=5.4424778761062
262 *109 14 2 -2 I(109,14)=5.716814159292 J(109,14)=-2
263 *109 99 1 17 I(109,99)=17 J(109,99)=9.283185840708
264 *110 22 2 0 I(110,22)=5.8407079646018 J(110,22)=0
265 *110 91 1 15 I(110,91)=15 J(110,91)=9.1592920353982
266 *111 53 1 9 I(111,53)=9 J(111,53)=5.070796460177
267 *111 60 2 6 I(111,60)=9.929203539823 J(111,60)=6
268 *112 15 2 1 I(112,15)=2.9823008849558 J(112,15)=1
269 *112 98 1 14 I(112,98)=14 J(112,98)=12.017699115044
270 *113 0 1 0 I(113,0)=0 J(113,0)=0
271 *113 113 2 15 I(113,113)=15 J(113,113)=15
272 *
273 *.....
274 *
275 *Plotting roots with coordinates:
276 *
277 *GPLOT XYML,X,Y / IND=M,1 POINT=[SwissB(30)],L OUTFILE=A
278 *.....
279 *GPLOT XYML,X,Y / IND=M,2 POINT=[SwissB(30)][RED],L INFILE=A OUTFILE=B
280 *.....
281 *
282 *Plotting parabolas:
283 *
284 *INFILE=B OUTFILE=C
285 *GPLOT Y(X)=(X^2-P*X+i*P)/15
286 *i=-2,42,1
287 *.....
288 *XSCALE=[Swiss(50)],0,20,40,60,80,113
289 *YSCALE=[Swiss(50)],0,20,40,60,80,113
290 *X=[RED],0,113,0.01
291 *
292 *INFILE=C OUTFILE=D
293 *GPLOT Y(X)=(-X^2+P*X+i*P)/15
294 *i=-27,6,1
295 *.....
296 *

```